

THE WEINSTEIN CONJECTURE IN PRODUCT OF SYMPLECTIC MANIFOLDS

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ABSTRACT. In this paper, using pseudo-holomorphic curve method, one proves the Weinstein conjecture in the product $P_1 \times P_2$ of two strongly geometrically bounded symplectic manifolds under some conditions with P_1 . In particular, if N is a closed manifold or a noncompact manifold of finite topological type, our result implies that the Weinstein conjecture in $\mathbb{CP}^2 \times T^*N$ holds.

Key words: Weinstein conjecture, J -holomorphic sphere, geometrically bounded.

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CONTENTS

1. Introduction	1
2. Product of Regular Almost Complex Structures	3
3. Holomorphic Spheres	6
4. Proof of Main Theorem	9
5. Applications	12
References	18

1. INTRODUCTION

Let M be a symplectic manifold with symplectic form ω . A hypersurface $S \subset M$ is said to be of contact type if there exists a vector field X defined on some neighborhood U of S such that (i) X is transversal to S and (ii) $L_X\omega = \omega$.

For any hypersurface S in symplectic manifold M , there exists a 1-dimensional characteristic line bundle $\mathcal{L}_S \subset TS$ defined by:

$$\mathcal{L}_S = \{(x, \xi) \in T_x S \mid \omega_x(\xi, \eta) = 0, \quad \forall \eta \in T_x S\}.$$

Let ξ be a section of the characteristic line bundle. The Weinstein conjecture claims that if S is a compact hypersurface of contact type, then S carries at least one closed orbit of ξ , see [21].

In 1987, C. Viterbo [19] proved the Weinstein conjecture for $(\mathbb{R}^{2n}, \omega_0)$ with the standard symplectic form ω_0 . Later H. Hofer and C. Viterbo [9] showed the Weinstein conjecture was true for $(T^*M, -d\lambda)$, where λ was the Liouville form on the cotangent bundle T^*M of a compact manifold M . A. Floer, H. Hofer and C.

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Viterbo [5] proved the stabilized Weinstein conjecture for $(P \times \mathbb{C}^l, \omega \oplus \omega_0)$ under the assumption $[\omega] = 0$ on $\pi_2(P)$. In 1992, H. Hofer and C. Viterbo [10] introduced the pseudo-holomorphic curve method into the study of the Weinstein conjecture for some cases where the holomorphic spheres appeared. They proved the Weinstein conjecture in \mathbb{CP}^n , $S^2 \times P$, if P was a compact symplectic manifold with some conditions. Lu [15] extended the results of H. Hofer and C. Viterbo to the strongly geometrically bounded (SGB) symplectic manifolds. He showed the Weinstein conjecture holds in $S^2 \times T^*N$, if N was a closed manifold or a noncompact manifold of finite topological type. G. Liu and G. Tian completely proved the stabilized version Weinstein conjecture in [12].

Since the product of regular almost complex structures is not regular in general (see [14]), the method of [10] can not be applied directly to any product manifolds. Making use of the regularity criterion in [17], we proved that there exists a regular almost complex structure, which is the product of regular almost complex structures, on the product of some 4-dimensional manifolds and symplectic manifolds. So this makes it possible to use the method of [10] to study the Weinstein conjecture for the product manifolds. In this paper, one proves the Weinstein conjecture in the product $P_1 \times P_2$ of two SGB symplectic manifolds under some conditions with P_1 . In particular, if N is a compact manifold or a noncompact manifold of finite topological type, our result implies that the Weinstein conjecture in $\mathbb{CP}^2 \times T^*N$ holds.

Next, we will introduce some notations and our result. Let (V, ω) be a symplectic manifold. Let $\mathcal{F}(V, \omega)$ be the space of all smooth almost complex structures which are compatible with ω on (V, ω) . The subset of regular almost complex structures (see Definition 2.1) in $\mathcal{F}(V, \omega)$ is denoted by $\mathcal{F}_{reg}(V, \omega)$. For $J \in \mathcal{F}(V, \omega)$, define $m(V, \omega, J)$ in $(0, +\infty]$ by

$$m = \inf\{\langle \omega, [u] \rangle \mid u \text{ is a nonconstant } J\text{-holomorphic sphere}\},$$

where $\langle \omega, [u] \rangle = \int_{S^2} u^* \omega$ which depends only on the free homotopy class $[u]$ of u . Define $m(V, \omega) \in [0, +\infty]$ by

$$m(V, \omega) = \inf\{\langle \omega, \alpha \rangle \mid \alpha \in [S^2, V], \langle \omega, \alpha \rangle > 0\},$$

where $[S^2, V]$ stands for the free homotopy classes. A homotopy class α is said to be ω -minimal if $m(V, \omega) = \langle \omega, \alpha \rangle$ and $\langle \omega, \alpha \rangle > 0$. Let α be an ω -minimal homotopy class such that there exists a $J \in \mathcal{F}(V, \omega)$ satisfies $m(V, \omega, J) = \langle \omega, \alpha \rangle$. Define $\mathcal{H}(\alpha, J, \Sigma_0, \Sigma_\infty)$ to be the set of all $u \in C^\infty(S^2, V)$ such that

$$(1) \quad [u] = \alpha, \quad u(*) \in \Sigma_*, \quad * \in \{0, \infty\}, \quad \int_{|z| \leq 1} u^* \omega = \frac{1}{2} \langle \omega, \alpha \rangle, \quad \bar{\partial}_J u = 0$$

where Σ_0, Σ_∞ are two disjoint smooth submanifolds of V and closed as subsets. We also assume that one of Σ_0 and Σ_∞ is compact.

Under certain conditions, there are almost complex structures $\{\tilde{J}\}$, which are as close as we want to J with respect to C^1 -topology, such that $\mathcal{H}(\alpha, \tilde{J}, \Sigma_0, \Sigma_\infty)$ is a smooth compact free S^1 -manifold. Such a \tilde{J} is called a regular almost complex structure at the situation $(\alpha, \Sigma_0, \Sigma_\infty)$. Moreover, for any given regular \tilde{J}_1 and \tilde{J}_2 which are close to J the compact smooth S^1 -manifolds \mathcal{H}_1 and \mathcal{H}_2 belong to the

same free S^1 -cobordism class. $d(\alpha, J, \Sigma_0, \Sigma_\infty) := [\mathcal{H}(\alpha, \tilde{J}, \Sigma_0, \Sigma_\infty)]$, where \tilde{J} is in a sufficiently small neighborhood of J . The definition of $d(\alpha, J, \Sigma_0, \Sigma_\infty)$ is not depend on the choice of \tilde{J} .

The following is our main result of this paper.

Theorem 1.1. *Let $(P_1, \omega_1), (P_2, \omega_2)$ be two SGB symplectic manifolds with $\dim P_1 = 4$. $\alpha_1 \in [S^2, P_1]$ is an ω_1 -minimal free homotopy class which can be represented by an embedded J_1 -holomorphic sphere such that*

$$0 < \langle \omega_1, \alpha_1 \rangle \leq m(P_2, \omega_2),$$

where $J_1 \in \mathcal{F}_{reg}(P_1, \omega_1)$. $\Sigma_0^1, \Sigma_\infty^1$ are two disjoint nonempty compact submanifolds of P_1 . Σ_0^2 is a nonempty compact submanifold of P_2 . Let $\Sigma_0 = \Sigma_0^1 \times \Sigma_0^2$, $\Sigma_\infty = \Sigma_\infty^1 \times P_2$. Suppose that there is a smooth Hamiltonian $H : P_1 \times P_2 \rightarrow \mathbb{R}$ such that

$$H|_{\mathcal{U}(\Sigma_0)} \equiv h_0, H|_{\mathcal{U}(\Sigma_\infty)} \equiv h_\infty, h_0 < h_\infty \text{ and } h_0 \leq H \leq h_\infty.$$

Where the open neighborhoods $\mathcal{U}(\Sigma_0)$ and $\mathcal{U}(\Sigma_\infty)$ are disjoint and such that

$$(2) \quad K := (P_1 \times P_2) \setminus (\mathcal{U}(\Sigma_0) \cup \mathcal{U}(\Sigma_\infty)) \text{ is compact.}$$

Then if $d(\alpha_1, J_1, \Sigma_0^1, \Sigma_\infty^1) \neq [\emptyset]$, the Hamiltonian system $\dot{x} = X_H(x)$ possesses a nonconstant T -periodic solution $x = x(t)$ with

$$0 < T(h_\infty - h_0) < \langle \omega_1, \alpha_1 \rangle, h_0 \leq H(x(t)) \leq h_\infty.$$

As in [10], it is easy to prove the Weinstein conjecture in $P_1 \times P_2$ from Theorem 1.1.

Corollary 1.2. *Let $(P_1 \times P_2, \omega_1 \oplus \omega_2)$, $\alpha_1 \in [S^2, P_1]$, $J_1 \in \mathcal{F}_{reg}(P_1, \omega_1)$, $\Sigma_0^1, \Sigma_\infty^1, \Sigma_0, \Sigma_\infty$ satisfy the hypothesis of Theorem 1.1. Then any stable compact smooth hypersurface \mathcal{S} in $(P_1 \times P_2, \omega_1 \oplus \omega_2)$ separating Σ_0 from Σ_∞ possesses at least one periodic Hamiltonian trajectory.*

Corollary 1.3. *Let $u : S^2 = \mathbb{C} \cup \{\infty\} \hookrightarrow \mathbb{CP}^2$ be a holomorphic embedding and $\{x\}, \{y\}$ two different points in the image of u . Suppose N is a closed manifold or a manifold of finite topological type. Define $\Sigma_0 = \{p_0\}$, $p_0 \in \{x\} \times T^*N$, $\Sigma_\infty = \{y\} \times T^*N$. Then any stable compact smooth hypersurface \mathcal{S} in $\mathbb{CP}^2 \times T^*N$ separating Σ_0 from Σ_∞ possesses at least one periodic Hamiltonian trajectory.*

2. PRODUCT OF REGULAR ALMOST COMPLEX STRUCTURES

Let (V, ω) be a symplectic manifold and $J \in \mathcal{F}(V, \omega)$. For a smooth map $u : S^2 \rightarrow V$, the space of smooth vector fields $\xi(z) \in T_{u(z)}V$ along u will be denoted by $\Omega^0(S^2, u^*TV)$ and the space of smooth J -antilinear 1-forms on S^2 with values in u^*TV by $\Omega^{0,1}(S^2, u^*TV)$. Then the vertical differential of $\bar{\partial}_J(u)$, $D_u : \Omega^0(S^2, u^*TV) \rightarrow \Omega^{0,1}(S^2, u^*TV)$, have the following expression:

$$(3) \quad D_u \xi = \frac{1}{2}(\nabla \xi + J(u) \nabla \xi \circ i) - \frac{1}{2}J(u)(\nabla_\xi J)(u) \partial_J(u), \quad \forall \xi \in \Omega^0(S^2, u^*TV)$$

where $\partial_J(u) := \frac{1}{2}(du - J \circ du \circ i)$ and ∇ denotes the Levi-Civita connection of the metric $\omega(\cdot, J\cdot)$.

A J -homomorphic sphere $u : S^2 \rightarrow V$ is said to be **multiply covered** if there exists a J -holomorphic sphere $u' : S^2 \rightarrow V$, and a holomorphic branched covering $\phi : S^2 \rightarrow S^2$ such that

$$u = u' \circ \phi, \quad \deg(\phi) > 1.$$

The curve u is called **simple** if it is not multiply covered.

Definition 2.1. An almost complex structure J on V is called *regular* at the situation $(\alpha, \Sigma_0, \Sigma_\infty)$, if for every $u \in H^{2,2}(S^2, V)$ which satisfies condition (1) D_u is onto. In particular for a regular J the set $\mathcal{H}(\alpha, J, \Sigma_0, \Sigma_\infty)$ is a smooth S^1 -manifold.

Remark: By elliptic regularity theory every $u \in H^{2,2}(S^2, V)$ which satisfies condition (1) is smooth.

There is a regularity criterion in [17] which is very important for us.

Lemma 2.2 (Lemma 3.3.2 in [17]). *Let $E \rightarrow S^2$ be a complex vector bundle of rank n and*

$$D : \Omega^0(S^2, E) \rightarrow \Omega^{0,1}(S^2, E)$$

be a real linear Cauchy-Riemann operator. Suppose that there exists a splitting $E = L_1 \oplus \cdots \oplus L_n$ into complex line bundles such that each subbundle $L_1 \oplus \cdots \oplus L_k$, $k = 1, \dots, n$, is invariant under D . Then D is surjective if and only if $c_1(L_k) \geq -1$ for every k .

Remark: $\Omega^0(S^2, E)$ denotes the space of all smooth vector fields $\xi(z) \in E_z$. $\Omega^{0,1}(S^2, E)$ denotes the space of smooth J -antilinear 1-forms on S^2 with values in E . Let $\pi_k : E \rightarrow L_k$ denote the projection onto the k th summand. Then the subbundle $L_1 \oplus \cdots \oplus L_k$ is invariant under D means that if $i > k$, $\pi_i(D\xi_j) = 0, \forall \xi_j \in \Omega^0(S^2, L_j)$, $j = 1, \dots, k$. Here and throughout this section we identify the first Chern class $c_1(L)$ of L with the corresponding Chern number $\langle c_1(L), [S^2] \rangle$.

Remark: The operator D_u is obviously a real linear Cauchy-Riemann operator.

Using Lemma 2.2, we can give a sufficient condition which guarantees a product regular almost complex structure is still regular. First, we will introduce some notations. The number of all self-intersections of a curve u will be denoted by

$$\delta(u) := \frac{1}{2} \# \{ (z_0, z_1) \in \Sigma \times \Sigma \mid z_0 \neq z_1, u(z_0) = u(z_1) \}.$$

We denote by $c_1(A) = \langle c_1(TM), A \rangle$ for $A \in H_2(M; \mathbb{Z})$, where $c_1(TM)$ is the first Chern class of TM , by $A_0 \cdot A_1$ the intersection number of two classes A_0 and A_1 , and by $\chi(\Sigma)$ the Euler characteristic of a closed Riemann surface Σ .

Lemma 2.3 (adjunction inequality in [17]). *Let (M, J) be an almost complex 4-manifold and $A \in H_2(M; \mathbb{Z})$ be a homology class that is represented by a simple J -holomorphic curve $u : \Sigma \rightarrow M$. Then*

$$2\delta(u) - \chi(\Sigma) \leq A \cdot A - c_1(A).$$

with equality if and only if u is an immersion with only transverse self-intersections (i.e. if $z_0 \neq z_1$ and $u(z_0) = u(z_1) =: x$, then $T_x M = \text{im} du(z_0) \oplus \text{im} du(z_1)$).

For the 4-manifolds, we have the following Proposition.

Proposition 2.4. *Let (P_1, ω_1) be a symplectic 4-manifold and (P_2, ω_2) a symplectic manifold. Assume $\alpha_1 \in [S^2, P_1]$ is an ω_1 -minimal free homotopy class which can be represented by an embedded J_1 -holomorphic sphere u such that*

$$0 < \langle \omega_1, \alpha_1 \rangle \leq m(P_2, \omega_2).$$

$\Sigma_0^1, \Sigma_\infty^1$ are two disjoint nonempty compact submanifolds of P_1 . Σ_0^2 is a nonempty compact submanifold of P_2 . Let $\Sigma_0 = \Sigma_0^1 \times \Sigma_0^2$, $\Sigma_\infty = \Sigma_\infty^1 \times P_2$ and $\alpha \in [S^2, V : z \mapsto (\alpha_1(z), p_0)]$, where $p_0 \in \Sigma_0^2$. If J_1 is regular at the situation $(\alpha_1, \Sigma_0^1, \Sigma_\infty^1)$ in P_1 and $J_2 \in \mathcal{F}(P_2, \omega_2)$, then the product almost complex structure $J = J_1 \times J_2$ is regular at the situation $(\alpha, \Sigma_0, \Sigma_\infty)$ in $P_1 \times P_2$.

Proof. First it is easy to see every J_1 -holomorphic sphere u which represents α_1 is simple. In fact, if u is multiply covered there exists a J_1 -holomorphic sphere $u' : S^2 \rightarrow P_1$, and a holomorphic branched covering $\phi : S^2 \rightarrow S^2$ such that

$$u = u' \circ \phi, \quad \deg(\phi) = k > 1.$$

Evidently $\langle \omega_1, [u'] \rangle = \frac{1}{k} \langle \omega_1, \alpha_1 \rangle$ since $\alpha_1 = [u]$. Hence

$$\langle \omega_1, \alpha_1 \rangle \leq m(P_1, \omega_1, J_1) \leq \frac{1}{k} \langle \omega_1, \alpha_1 \rangle,$$

giving a contradiction to our assumption that α_1 is ω_1 -minimal. Assume u represents the homology class $A \in H^2(P_1; \mathbb{Z})$, i.e. $u_*([S^2]) = A$. Then all the J_1 -holomorphic spheres represent α_1 will represent A .

Since $A \in H^2(P_1; \mathbb{Z})$ is represented by an embedded J_1 -holomorphic sphere u which is also simple, by the adjunction inequality we can get

$$-2 = A \cdot A - c_1(A).$$

For every simple J_1 -holomorphic sphere $v : S^2 \rightarrow P_1$ which represents A , we have

$$2\delta(v) - 2 \leq A \cdot A - c_1(A),$$

$$2\delta(v) \leq 0,$$

$$\delta(v) = 0.$$

The equality of the adjunction inequality holds for v . Thus every simple J_1 -holomorphic sphere v which represents A is an embedded curve. We can get every J_1 -holomorphic sphere represents α_1 is an embedded curve.

Assume $\tilde{u} \in H^{2,2}(S^2, P_1 \times P_2)$ and \tilde{u} satisfies

$$[\tilde{u}] = \alpha, \quad \tilde{u}(*) \in \Sigma_*, \quad * \in \{0, \infty\}, \quad \int_{|z| \leq 1} \tilde{u}^* \omega = \frac{1}{2} \langle \omega, \alpha \rangle, \quad \bar{\partial}_{J\tilde{u}} \tilde{u} = 0,$$

where $\omega = \omega_1 \oplus \omega_2$. The J -holomorphic α sphere has the form $\tilde{u}(z) = (u(z), p_0)$, where $u \in H^{2,2}(S^2, P_1)$ and satisfies

$$[u] = \alpha_1, \quad u(*) \in \Sigma_*^1, \quad * \in \{0, \infty\}, \quad \int_{|z| \leq 1} u^* \omega_1 = \frac{1}{2} \langle \omega_1, \alpha_1 \rangle, \quad \bar{\partial}_{J_1} u = 0.$$

We have the splitting

$$\begin{aligned}\tilde{u}^*T(P_1 \times P_2) &= u^*TP_1 \oplus (S^2 \times T_{p_0}P_2) \\ &= u^*TP_1 \oplus L_2 \oplus \dots \oplus L_{n+1}.\end{aligned}$$

It follows from the definition of D_u (3) that

$$D_u(du \circ \zeta) = du \circ \bar{\partial}_j \zeta$$

for every vector field $\zeta \in \text{Vect}(S^2)$. For the embedded curve u , the complex subbundle

$$L_0 := \text{im}(du) \subset u^*TP_1$$

is invariant under D_u . Now let $L_1 \subset u^*TP_1$ be the orthogonal complement of L_0 with respect to any Hermitian inner product of u^*TP_1 . Then by Lemma 2.2

$$u^*TP_1 = L_0 \oplus L_1, \quad c_1(L_0) \geq -1, \quad c_1(L_1) \geq -1,$$

because J_1 is regular at the situation $(\alpha_1, \Sigma_0^1, \Sigma_\infty^1)$ in P_1 . In the product manifold $(P_1 \times P_2, \omega_1 \oplus \omega_2)$, $\omega_1(\cdot, J_1 \cdot) + \omega_2(\cdot, J_2 \cdot)$ defines a product metric on $P_1 \times P_2$. Let ∇ be the Levi-Civita connection on $P_1 \times P_2$ and ∇^i the Levi-Civita connection on P_i , $i = 1, 2$, respectively. By the relation between ∇ and ∇^i , $i = 1, 2$, we know in the product manifold $P_1 \times P_2$

$$D_{\tilde{u}}\xi_j = D_u\xi_j, \quad \forall \xi_j \in \Omega^0(S^2, L_j), j = 0, 1.$$

Thus the subbundles $L_0, L_0 \oplus L_1$, are invariant under $D_{\tilde{u}}$ too. In the trivial bundle $S^2 \times T_{p_0}P_2$, each subbundle $L_2 \oplus \dots \oplus L_{1+j}$, $j = 1, \dots, n$, is obviously invariant under $D_{\tilde{u}}$. $c_1(L_j) \geq -1$, $j = 2, \dots, n+1$. By Lemma 2.2 again, we know $D_{\tilde{u}}$ is surjective. \square

Remark: From the arguments of Lemma 3.3.3, Corollary 3.3.4 and Corollary 3.3.5 in [17], we can get the above Proposition easily.

3. HOLOMORPHIC SPHERES

Let us recall the definition of geometrically bounded manifold (cf.[2], [7], [15]).

Definition 3.1. Let (M, ω) be a symplectic manifold without boundary. we will call it geometrically bounded if there exists an almost complex structure J and a complete Riemannian metric g on M such that the following properties are satisfied:

1. J is uniformly tamed by ω ; that is, there exist strictly positive constants α and β such that

$$\omega(X, JX) \geq \alpha \|X\|_g^2 \quad \text{and} \quad |\omega(X, Y)| \leq \beta \|X\|_g \|Y\|_g$$

for all $X, Y \in TM$;

2. the sectional curvature $K_g \leq C$ (a positive constant) and the injectivity radius $i(M, g) > 0$.

Definition 3.2 (Definition 2.4 in [15]). In Definition 3.1 if we require $J \in \mathcal{F}(M, \omega)$, then the symplectic manifold (M, ω) is called strongly geometrically bounded (SGB).

It is well known that the closed symplectic manifolds are SGB and a product of two SGB symplectic manifolds is SGB. It is easy to prove the symplectic manifolds which at infinity are isomorphic to the symplectization of a closed contact manifold are SGB (cf. [4]). The standard cotangent bundles as well as the twisted cotangent bundles over closed manifolds are SGB (cf. [4], [15]).

Let $(P_1, \omega_1, J_1, g_1), (P_2, \omega_2, J_2, g_2)$ be two SGB symplectic manifolds such that $\dim P_1 = 4$. $V = P_1 \times P_2$, $\omega = \omega_1 \oplus \omega_2$, $J = J_1 \times J_2$, $g = g_1 \oplus g_2$. Then (V, ω, J, g) is a SGB symplectic manifold. Assume $m(V, \omega, J) < \infty$, and let $\alpha \in [S^2, V : z \mapsto (\alpha_1(z), p_0)]$, $p_0 \in \Sigma_0^2$, be a free homotopy class which is defined in Proposition 2.4 such that

$$(4) \quad \langle \omega, \alpha \rangle = m(V, \omega, J).$$

From the definition of $m(V, \omega, J)$, we can get that a J -holomorphic sphere which represents α is simple.

Consider the Banach manifold \mathcal{B} consisting of all maps $u \in H^{2,2}(S^2, V)$ such that with $D = \{z \mid |z| \leq 1\}$

$$[u] = \alpha, \quad u(*) \in \Sigma_*, \quad * \in \{0, \infty\}, \quad \int_D u^* \omega = \frac{1}{2} \langle \omega, \alpha \rangle,$$

where Σ_0, Σ_∞ are two disjoint smooth submanifolds without boundary of V and closed as subsets in V . We also assume that one of Σ_0 and Σ_∞ is compact. Denote by $\bar{X}_J \rightarrow S^2 \times V$ the vector bundle whose fiber over $(z, v) \in S^2 \times V$ consists of all linear maps $\phi : T_z S^2 \rightarrow T_v V$ such that $J(v)\phi = -\phi \circ i$. Given $u : S^2 \rightarrow V$ we denote by $\bar{u} : S^2 \rightarrow S^2 \times V$ the "graph map" $\bar{u}(z) = (z, u(z))$ and write $\bar{u}^* \bar{X}_J \rightarrow S^2$ for the pull back bundle. Let \mathcal{E} be the Banach bundle $\mathcal{E} \rightarrow \mathcal{B}$ whose fiber $\mathcal{E}_u = H^{1,2}(\bar{u}^* \bar{X}_J)$ at $u \in H^{2,2}(S^2, V)$ consists of all $H^{1,2}$ sections of $\bar{u}^* \bar{X}_J \rightarrow S^2$. The nonlinear Cauchy Riemann operator $\bar{\partial}_J$, $\bar{\partial}_J u = du + J \circ du \circ i$, can be considered as a smooth section of $\mathcal{E} \rightarrow \mathcal{B}$, and its zero set is $\mathcal{H}(\alpha, J, \Sigma_0, \Sigma_\infty)$. By elliptic regularity theory every $u \in \mathcal{B}$ with $\bar{\partial}_J u = 0$ is smooth. H. Hofer and C. Viterbo proved some propositions-Propositions 2.3, 2.4 and 2.7 in [10]- for the compact manifold V which guaranteed the d-index was well defined and made the existence of closed orbit possible. Lu proved a prior compactness property (Proposition 2.5 in [15]) for the SGB symplectic manifold. Utilizing the prior compactness and the assumption (2), Lu [15] showed the Propositions 2.3 and 2.4 in [10] also held true for the case of SGB symplectic manifold if the neighborhood U_J and $\mathcal{F}_{reg}(V, \omega) \cap U_J$ of J in these Propositions were replaced by $\mathcal{U}(J, \delta, f_{r_0})$ and $\mathcal{F}_{reg}(V, \omega) \cap \mathcal{U}(J, \delta, f_{r_0})$. The definition of $\mathcal{U}(J, \delta, f_{r_0})$ is given in [15]. In the following, $\mathcal{U}(J, \delta, f_{r_0})$ is abbreviated to \mathcal{U} . So the d-index $d(\alpha, J, \Sigma_0, \Sigma_\infty) := [\mathcal{H}(\alpha, \tilde{J}, \Sigma_0, \Sigma_\infty)]$ is well defined in the SGB symplectic manifold.

Proposition 3.3. *Let (V, ω) be a SGB symplectic manifold, $J \in \mathcal{F}(V, \omega)$, $m(V, \omega, J) = \langle \omega, \alpha \rangle$, Let Σ_0, Σ_∞ be described above, then there exists an open neighborhood \mathcal{U} of J such that*

- (1) *For all $\tilde{J} \in \mathcal{F}_{reg}(V, \omega) \cap \mathcal{U}$, the set $\mathcal{H}(\alpha, \tilde{J}, \Sigma_0, \Sigma_\infty)$ is a compact smooth S^1 -manifold.*
- (2) *$\mathcal{F}_{reg}(V, \omega) \cap \mathcal{U}$ is dense in \mathcal{U} .*

(3) Let J'_0 and J'_1 be close to J in \mathcal{U} , and $J'_0, J'_1 \in \mathcal{F}_{reg}(V, \omega)$. Suppose $\lambda \rightarrow J'_\lambda$ is a smooth homotopy with $\lambda \in [0, 1]$ and $J'_\lambda \in \mathcal{U}$. Then there exists a smooth arbitrarily small perturbation of $[\lambda \rightarrow J'_\lambda]$ with the end points fixed, say $[\lambda \rightarrow \tilde{J}_\lambda]$, such that

$$\mathcal{M} := \{(\lambda, u) \in [0, 1] \times \mathcal{B} \mid \bar{\partial}_{\tilde{J}_\lambda} u = 0\}$$

is a compact S^1 -manifold with boundary

$$\partial \mathcal{M} = \mathcal{H}(\alpha, J'_0, \Sigma_0, \Sigma_\infty) \amalg \mathcal{H}(\alpha, J'_1, \Sigma_0, \Sigma_\infty).$$

Let $H : V \rightarrow \mathbb{R}$ be a smooth map and $g_J(\cdot, \cdot) = \omega(\cdot, J \cdot)$ the Riemannian metric. We denote by ∇H the gradient of H with respect to the metric g_J . For suitable neighborhoods $\mathcal{U}(\Sigma_0), \mathcal{U}(\Sigma_\infty)$ of Σ_0, Σ_∞ respectively, suppose $H|_{\mathcal{U}(\Sigma_0)} \equiv h_0$, $H|_{\mathcal{U}(\Sigma_\infty)} \equiv h_\infty$, $h_0 < h_\infty$ and $h_0 \leq H \leq h_\infty$. Consider the subset

$$W = [(S^2 \setminus \{0, \infty\}) \times V] \cup [\{0\} \times \mathcal{U}(\Sigma_0)] \cup [\{\infty\} \times \mathcal{U}(\Sigma_\infty)].$$

We define a section \hat{h} of $\bar{X}_J|_W$ associated to H by

$$\hat{h} : W \rightarrow \bar{X}_J, \quad \hat{h}(z, v) =: \phi.$$

Where ϕ is the unique complex antilinear map $T_z S^2 \rightarrow T_v V$ satisfying the following:

1. If $z = 0$ or ∞ , ϕ is the zero map,
2. If $z \neq 0$ and $z \neq \infty$, ϕ maps the tangent vector $z \in T_z S^2 = \mathbb{C}$ to $\frac{1}{2\pi} \nabla H(v)$. Here we took the identity chart $S^2 \supset \mathbb{C} \simeq \mathbb{C}$ to distinguish in $T_z S^2$ for $z \in \mathbb{C}$ the tangent vector z .

If $u \in \mathcal{B}$ then the associated graph map \bar{u} , $\bar{u}(z) = (z, u(z))$ maps $z \in S^2$ into $W \subset S^2 \times V$. Consequently we can define $h(u) \in \mathcal{E}$ by

$$h(u)(z) = \hat{h}(z, u(z)).$$

Now we define a parameter depending family of smooth section of $\mathcal{E} \rightarrow \mathcal{B}$ by

$$f_\lambda(u) = \bar{\partial}_J u + \lambda h(u).$$

Clearly, f_λ is S^1 -equivalent for every λ and f_λ is a Fredholm section in the sense that at every zero u of f_λ the linearisation $Df_\lambda : T_u \mathcal{B} \rightarrow \mathcal{E}_u$ is Fredholm. Consider the set

$$\mathcal{C} = \{(\lambda, u) \in [0, +\infty) \times \mathcal{B} \mid f_\lambda(u) = 0\}.$$

By elliptic regularity theory, $\mathcal{C} \subset [0, +\infty) \times C^\infty(S^2, V)$. let $\mathcal{C}_\lambda = \{u \mid (\lambda, u) \in \mathcal{C}\}$. Then \mathcal{C}_0 is a compact smooth manifold with a free smooth S^1 -action, and $\mathcal{C}_0 = \mathcal{H}(\alpha, J, \Sigma_0, \Sigma_\infty)$. Lu [15] showed that if the manifold V is SGB, the Proposition 2.7 in [10] was also true.

Proposition 3.4 (PROPOSITION 3.1 in [15]). *Let $\alpha \in [S^2, V]$, $\Sigma_0, \Sigma_\infty, J$ and H be as above, and let \mathcal{C} be compact. Then*

$$d(\alpha, J, \Sigma_0, \Sigma_\infty) = [\emptyset],$$

i.e. $\mathcal{H}(\alpha, J, \Sigma_0, \Sigma_\infty)$ is the boundary of a smooth compact manifold \mathcal{M} equipped with a free S^1 -action, so that the action on $\partial \mathcal{M}$ coincides with the action on \mathcal{H} .

As in [10] and [15], we have the following Proposition:

Proposition 3.5. *Let (V, ω) be a SGB symplectic manifold. Σ_0, Σ_∞ are described above. $J \in \mathcal{F}(V, \omega)$ such that $m(V, \omega, J) \geq \langle \omega, \alpha \rangle$, where $\alpha \in [S^2, V]$. Let $\mathcal{E} \rightarrow \mathcal{B}$ be the Hilbert space bundle defined above. Let $H : V \rightarrow \mathbb{R}$ be a smooth map such that*

$$H|_{\mathcal{U}(\Sigma_0)} \equiv h_0, H|_{\mathcal{U}(\Sigma_\infty)} \equiv h_\infty, h_0 < h_\infty \text{ and } h_0 \leq H \leq h_\infty.$$

Let \mathcal{C} be defined above. Then

- (1) If $(\lambda, u) \in \mathcal{C}$, then $\lambda \in [0, \lambda_\infty], \lambda_\infty = (h_\infty - h_0)^{-1} \langle \omega, \alpha \rangle$;
- (2) For every multi index β there is a constant $C_\beta > 0$ such that for every $(\lambda, u) \in \mathcal{C}$, $v = u \circ \phi$, here $\phi : S^1 \times \mathbb{R} \rightarrow \mathbb{C}, \phi(t, s) = e^{2\pi(s+it)}$.

$$|(D^\beta v)(x)| \leq C_\beta, \quad \forall x \in S^1 \times \mathbb{R}$$

- (3) There exists $\varepsilon > 0$ such that for every $(\lambda, u) \in \mathcal{C}$ we have: if $v(s)(S^1) \not\subset \mathcal{U}(\Sigma_0)$ then

$$\int_{-\infty}^s \int_0^1 v^* \omega - \lambda \int_0^1 H(v(s)(t)) dt \geq \varepsilon - \lambda h_0.$$

If $v(s)(S^1) \not\subset \mathcal{U}(\Sigma_\infty)$ then

$$\int_{-\infty}^s \int_0^1 v^* \omega - \lambda \int_0^1 H(v(s)(t)) dt \leq \langle \omega, \alpha \rangle - \varepsilon - \lambda h_\infty.$$

Sketch of the proof. From Theorem 2.9 in [15], we obtain that $\cup_{(\lambda, u) \in \mathcal{C}} u(S^2)$ is contained in a compact subset of V . Following almost the same arguments of Theorem 3.4 in [10], we can see that the proposition is also true. \square

4. PROOF OF MAIN THEOREM

The J -holomorphic sphere method always requires the regular almost complex structure. In order to get the relation between the d -index of $P_1 \times P_2$ with the d -index of P_i , $i \in \{1, 2\}$, we need a regular almost complex structure $J = J_1 \times J_2$, where $J_i \in \mathcal{F}_{reg}(P_i, \omega_i)$, $i \in \{1, 2\}$. However, the product of regular almost complex structures is not regular in general. Thus Proposition 2.4 is necessary for our case. Now we can prove Theorem 1.1.

Proof of Theorem 1.1. Let $\alpha \in [S^2, P_1 \times P_2]$ be of the form $[S^2 \rightarrow P_1 \times P_2 : z \mapsto (\alpha_1(z), p_0)]$, where $p_0 \in \Sigma_0^2$ is a fixed point and $\alpha_1 \in [S^2, P_1]$ is defined in the hypothesis of Theorem 1.1. On $P_1 \times P_2$ we take the product almost complex structure $J = J_1 \times J_2$, where $J_1 \in \mathcal{F}_{reg}(P_1, \omega_1), J_2 \in \mathcal{F}_{reg}(P_2, \omega_2)$. Then

$$\begin{aligned} \langle \omega_1 \oplus \omega_2, \alpha \rangle &= m(P_1 \times P_2, \omega_1 \oplus \omega_2, J) \\ &= m(P_1, \omega_1, J_1) = \langle \omega_1, \alpha_1 \rangle \leq m(P_2, \omega_2). \end{aligned}$$

By Proposition 2.4, J is regular at the situation $(\alpha, \Sigma_0, \Sigma_\infty)$. From $d(\alpha_1, J_1, \Sigma_0^1, \Sigma_\infty^1) \neq [\emptyset]$ and $m(P_1 \times P_2, \omega_1 \oplus \omega_2, J) \leq m(P_2, \omega_2)$, we have $d(\alpha, J, \Sigma_0, \Sigma_\infty) \neq [\emptyset]$.

In the following, we use the idea of [10] to prove Theorem 1.1. From Proposition 3.4, we can get \mathcal{C} is noncompact. We can assume $\{(\lambda_k, u_k)\} \subset \mathcal{C}$ such that

$$\lambda_k \rightarrow \lambda, \{(\lambda_k, u_k)\} \text{ has no convergent subsequence.}$$

For every $(\lambda, u) \in \mathcal{C}$, We define $v = u \circ \phi$, where $\phi : S^1 \times \mathbb{R} \rightarrow \mathbb{C}, \phi(t, s) = e^{2\pi(s+it)}$. Define $a : \mathbb{R} \rightarrow \mathbb{R}$ as follows, $a(s) := \int_{(-\infty, s] \times S^1} v^* \omega - \int_0^1 \lambda H(v(s, t)) dt$, where $\omega = \omega_1 \oplus \omega_2$. From Proposition 3.5, we have

$$\begin{aligned} \langle \omega_1, \alpha_1 \rangle - \lambda(h_\infty - h_0) &= \int_{-\infty}^{\infty} a'(s) ds \\ &= \int_{-\infty}^{+\infty} \int_0^1 |v_s|^2 ds dt \\ &\geq \int_{-\infty}^{s_0(v)} \int_0^1 v^* \omega \geq \varepsilon, \end{aligned}$$

where $s_0(v) = \sup\{s | v((-\infty, s] \times S^1) \subset \mathcal{U}(\Sigma)\}$. The last inequality is proved by the Lemma 3.1 in [10], which is also true here.

If $(\lambda, u) \in \mathcal{C}$, then

$$\begin{aligned} 0 &\leq \lambda \leq (h_\infty - h_0)^{-1}(\langle \omega_1, \alpha_1 \rangle - \varepsilon) \\ &=: (h_\infty - h_0)^{-1} \langle \omega_1, \alpha_1 \rangle - \varepsilon' \\ &= \lambda_\infty - \varepsilon'. \end{aligned}$$

We define two sequences of numbers by

$$\begin{aligned} s_k^0 &= \sup\{s | v_k((-\infty, s] \times S^1) \subset \mathcal{U}(\Sigma_0)\}, \\ s_k^\infty &= \inf\{s | v_k([s, +\infty) \times S^1) \subset \mathcal{U}(\Sigma_\infty)\}. \end{aligned}$$

Note that v_k denotes the map induced by u_k on the cylinder. Clearly $s_k^0 \leq s_k^\infty$.

Now we will show $s_k^\infty - s_k^0 \rightarrow \infty$. Arguing indirectly we may assume after taking a subsequence that for some constant $b > 0$

$$|s_k^\infty - s_k^0| \leq b \quad \text{for all } k.$$

Let $\hat{u}_k(z) = u_k(s_k z)$, $s_k > 0$. Replacing u_k by \hat{u}_k , we may assume that for some positive constant $c > 0$,

$$(5) \quad -c \leq \hat{s}_k^0 \leq \hat{s}_k^\infty \leq c,$$

where $\hat{s}_k^0, \hat{s}_k^\infty$ are the sequences associated to \hat{u}_k . From (5) and the previous discussion, it follows immediately that $\{\hat{u}_k\}$ has a convergent subsequence in $H^{2,2}(S^2, P_1 \times P_2)$, say $\hat{u}_k \rightarrow u$, where u satisfies

$$(6) \quad \begin{cases} \bar{\partial}_J u + \lambda h(u) = 0 \\ [u] = \alpha \\ u(0) \in \Sigma_0 \\ u(\infty) \in \Sigma_\infty. \end{cases}$$

In fact, since (5) holds, the nonlinearity $u \rightarrow h(u)$ is well behaved and one can use *Bubble off analysis* to obtain the solution u of (6).

$$\frac{1}{2} \langle \omega_1 \oplus \omega_2, \alpha \rangle = \frac{1}{2} \langle \omega_1, \alpha_1 \rangle = \int_D u_k^* \omega = \int_{s_k^{-1}D} \hat{u}_k^* \omega,$$

where $\omega = \omega_1 \oplus \omega_2$. If $s_k \rightarrow 0$ or $+\infty$, since $\hat{u}_k^* \omega \rightarrow u^* \omega$ in $H^{1,1}(S^2, \mathbb{R})$, we have

$$\frac{1}{2} \langle \omega_1, \alpha_1 \rangle = 0, \quad \text{or} \quad \frac{1}{2} \langle \omega_1, \alpha_1 \rangle = \langle \omega_1, \alpha_1 \rangle.$$

This contradiction shows that $s_k \in (a, \frac{1}{a})$ for all k for some suitable $a > 0$ independent of k . Hence, from the definition of \hat{u}_k and the fact that $\hat{u}_k \rightarrow u$ it follows that $\{u_k\}$ is convergent itself. However, this contradicts our assumption on $\{(\lambda_k, u_k)\}$. Therefore we know that

$$s_k^\infty - s_k^0 \rightarrow +\infty.$$

We have

$$\int_{s_k^0}^{s_k^\infty} \int_0^1 \left| -J(v_k) \frac{\partial v_k}{\partial t} - \lambda_k \nabla H(v_k) \right|^2 dt ds \leq \langle \omega_1, \alpha_1 \rangle - 2\varepsilon - \lambda_k(h_\infty - h_0).$$

Hence, we can find a sequence $\{s_k\}$,

$$s_k \in [s_k^0, s_k^\infty],$$

such that with $x_k := v_k(s_k, \cdot)$

$$\| -J(x_k) \dot{x}_k - \lambda_k \nabla H(x_k) \|_{L^2(x_k^* T(P_1 \times P_2))} \rightarrow 0.$$

Eventually taking a subsequence we may assume

$$(7) \quad \begin{cases} x_k \rightarrow x & \text{in } H^1(S^1, P_1 \times P_2) \\ -J(x) \dot{x} - \lambda \nabla H(x) = 0. \end{cases}$$

It is obvious that $x \in C^\infty(S^1, P_1 \times P_2)$. We first assume $\lambda = 0$. Let $\tilde{u}_k : S^2 \rightarrow P_1$ be the map induced from $u_k : S^2 \rightarrow P_1 \times P_2$ by the projection onto the first factor. Then,

$$\langle \omega_1, \alpha_1 \rangle = \int_{S^2} u_k^* \omega_1.$$

Now let $\tilde{v}_k : Z \rightarrow P_1$ be the map induced from \tilde{u}_k in the cylinder. Since ∇H vanishes on Σ_0 and Σ_∞ , \tilde{u}_k is holomorphic in the neighbourhood of all z such that $\tilde{u}_k(z)$ is close to Σ_0^1 or Σ_∞^1 .

If (7) holds, we can use $\tilde{v}_k : (-\infty, s_k] \times S^1 \rightarrow P_1$ and $\tilde{v}_k : [s_k, +\infty) \times S^1 \rightarrow P_1$ to construct maps

$$g_{\pm\infty}^k : S^2 \rightarrow P_1$$

such that

$$\begin{aligned} \langle \omega_1, \alpha_1 \rangle &\geq \liminf \left[\int_{S^2} (g_{+\infty}^k)^* \omega_1 \right] + \liminf \left[\int_{S^2} (g_{-\infty}^k)^* \omega_1 \right] \\ &\geq 2 \langle \omega_1, \alpha_1 \rangle. \end{aligned}$$

Since $\langle \omega_1, \alpha_1 \rangle > 0$, we have a contradiction. So we must have

$$\lambda_k \rightarrow \lambda \in (0, \lambda_\infty - \varepsilon'] \subset (0, \lambda_\infty)$$

and (7) still holds.

In the following, we will show that x is nonconstant. Arguing indirectly let us assume $x \equiv \text{const} \in P_1 \times P_2$. Denote by v_k^1 the P_1 -component of v_k . If

$$\int_{-\infty}^{s_k} \int_0^1 (v_k^1)^* \omega_1 \rightarrow 0,$$

we have $x = m_0 \in \Sigma_0$ and $v_k^1 \rightarrow m_0^1 \in \Sigma_0^1$ uniformly. Since $h|_{\mathcal{U}(\Sigma_0)} = 0$, this contradicts the definition of s_k^0 . Similarly,

$$\int_{s_k}^{+\infty} \int_0^1 (v_k^1)^* \omega_1 \rightarrow 0$$

is also impossible. Therefore, we have for some $\tau > 0$

$$(8) \quad \begin{aligned} & \int_{s_k}^{+\infty} \int_0^1 (v_k^1)^* \omega_1 \geq \tau, \\ & \int_{-\infty}^{s_k} \int_0^1 (v_k^1)^* \omega_1 \geq \tau, \\ & \int_{-\infty}^{\infty} \int_0^1 (v_k^1)^* \omega_1 = \langle \omega_1, \alpha_1 \rangle. \end{aligned}$$

Of course, since $v_k^1(\{s_k\} \times S^1)$ converges to a constant x^1 , by our assumption the first two integrals in (8) must be bounded below by $\langle \omega_1, \alpha_1 \rangle$ contradicting the equation

$$\int_{-\infty}^{\infty} \int_0^1 (v_k^1)^* \omega_1 = \langle \omega_1, \alpha_1 \rangle.$$

This shows that x has to be nonconstant. Eventually we have $H(x(t)) \in (h_0, h_\infty)$. This proves the theorem. \square

5. APPLICATIONS

We will give some applications of Theorem 1.1 in this section. Note that given the standard complex structure i on \mathbb{CP}^2 any two different points determine up to Möbius transformation a unique holomorphic sphere u . There is an embedding $u : S^2 = \mathbb{C} \cup \{\infty\} \hookrightarrow \mathbb{CP}^2$ which is holomorphic. Let $\Sigma_0^1 = \{x\}$ and $\Sigma_\infty^1 = \{y\}$, where x, y are different points in $u(S^2)$. Then with $\alpha_1 = [u]$, where $u(S^2)$ is the holomorphic curve running through x and y we have

$$d(\alpha_1, i, \Sigma_0^1, \Sigma_\infty^1) = [S^1] \neq [\emptyset].$$

We note here that i is a regular complex structure. Now let $P_1 = \mathbb{CP}^2$, P_2 be a SGB symplectic manifold with $[\omega_2]|_{\pi_2(P_2)} = 0$, $\Sigma_0 = \{p_0\}$, $p_0 \in \{x\} \times P_2$, $\Sigma_\infty = \{y\} \times P_2$. As an application of Theorem 1.1, we get the following corollary:

Corollary 5.1. *Let $\Sigma_0, \Sigma_\infty, P_2$ be as above, then any stable compact smooth hypersurface \mathcal{S} in $\mathbb{CP}^2 \times P_2$ separating Σ_0 from Σ_∞ possesses at least one periodic Hamiltonian trajectory.*

It is well known that the standard cotangent bundles (T^*N, ω) over closed manifolds N is SGB with $[\omega]|_{\pi_2(T^*N)} = 0$ (cf. [4], [15]).

Liouville manifold $(\hat{M}, \hat{\lambda})$ is a SGB symplectic manifold with $[d\hat{\lambda}]|_{\pi_2(\hat{M})} = 0$. Let us recall the definition of Liouville manifold now. A 1-form α on a manifold Σ is called a contact form for $\xi := \ker \alpha$, if $d\alpha$ is nondegenerate on ξ . In this case ξ is called a contact structure. A compact exact symplectic manifold with boundary (M, λ) is called a **Liouville domain**, if $(\Sigma := \partial M, \alpha := \lambda|_{\partial M})$ is a contact submanifold. We know every Liouville domain carries a Liouville vector field X defined by $\iota_X \omega = \lambda$, and the contact condition implies that X points outward at the boundary. We can paste the positive end of a symplectization $(\Sigma \times [0, \infty), d(e^t \alpha))$ along the boundary Σ . Then we obtain a complete **Liouville manifold**, which is denoted by $(\hat{M}, \hat{\lambda})$.

As in [1], [20], we introduce the following notation.

Definition 5.2. *A noncompact manifold M is said to be of finite topological type, if there is a compact domain $\Omega \subset M$ such that $M \setminus \overset{\circ}{\Omega}$ is diffeomorphic to $\partial\Omega \times [1, \infty)$.*

Actually, if M is a subset of a closed manifold or if M is of finite topological type the cotangent bundles (T^*M, ω) with standard symplectic structure are geometrically bounded. This is first pointed out by Audin, Lalonde and Polterovich [2] P.286. Lu [15] also claimed the cotangent bundle of a finite topological type manifold with twisted symplectic structure is SGB and omit the proof. In the following, we will give a proof of this for the completeness of our results. Our proof uses the idea of Proposition 2.2 in [4].

Proposition 5.3. *Let M be a manifold of finite topological type, then the cotangent bundle (T^*M, ω) with standard symplectic structure is SGB.*

Proof. Since M is of finite topological type, we may assume there is a compact domain $\Omega \subset M$ such that $M \setminus \overset{\circ}{\Omega}$ is diffeomorphic to $\partial\Omega \times [1, \infty)$. Assume the diffeomorphism is $h : \partial\Omega \times [1, \infty) \rightarrow M \setminus \overset{\circ}{\Omega}$. Denote $\Lambda = M \setminus \overset{\circ}{\Omega}$, $\Lambda_s = h(\partial\Omega \times [s, \infty))$, $s \geq 1$, $\partial\Lambda_s = h(\partial\Omega \times \{s\})$.

First we will define the Riemannian metric on T^*M . Let φ_t be the flow on T^*M formed by fiberwise dilations by the factor e^t . Choose a fiberwise convex hypersurface $\Sigma \subset T^*M|_{\Omega}$, enclosing the compact domain Ω . Note that Σ has contact type for ω . Let U be the closure of the unbounded part of the complement to Σ in $T^*M|_{\Omega}$. Then $U = \cup_{t \geq 0} \varphi_t(\Sigma)$. On the closure of the bounded part of the complement to Σ in $T^*M|_{\Omega}$, we can choose a compatible almost complex structure J . Let g be the Riemannian metric determined by ω and J , i.e. $g(\cdot, \cdot) = \omega(\cdot, J\cdot)$. (We also require that the radical vector is g -orthogonal to Σ .) Now we can extend these structures to U so that

$$(9) \quad \varphi_t^* g = e^t g \quad \text{for } t \geq 0,$$

i.e. g , just as ω , is homogeneous of degree one with respect to the dilations, and

$$J \circ (\varphi_t)_* = (\varphi_t)_* \circ J.$$

Then the metric g , the almost complex structure J and the standard symplecture ω are compatible on U . Hence are compatible on $T^*M|_{\Omega}$. To define the Riemannian

metric on $T^*M|_\Lambda$, let $\bar{\psi}_s$ be the flow of the vector field ∂_s on $\partial\Omega \times [1, \infty)$, i.e.

$$\begin{aligned}\bar{\psi}_s : \quad \partial\Omega \times [1, \infty) &\rightarrow \partial\Omega \times [1 + s, \infty) \\ (x, s_0) &\mapsto (x, s_0 + s).\end{aligned}$$

Let $\psi_s = h \circ \bar{\psi}_s \circ h^{-1} : \Lambda \rightarrow \Lambda_{1+s}$. Then there is a natural symplectomorphism which lifts ψ_s (see [3] Chapter 2)

$$\begin{aligned}\psi_{s\sharp} : T^*M|_\Lambda &\rightarrow T^*M|_{\Lambda_{1+s}} \\ (x, \xi) &\mapsto (\psi_s(x), (\psi_s^{-1})^*\xi).\end{aligned}$$

It is easy to see

$$(10) \quad \psi_{s\sharp} \circ \varphi_t = \varphi_t \circ \psi_{s\sharp}.$$

Now extend those structures to $T^*M|_\Lambda$ so that

$$(11) \quad (\psi_{s\sharp})^*g = g \quad s \geq 1$$

and

$$J \circ (\psi_{s\sharp})_* = (\psi_{s\sharp})_* \circ J.$$

We know the standard symplectic structure ω also satisfies $(\psi_{s\sharp})^*\omega = \omega$. Then ω, J , and g are compatible on $T^*M|_\Lambda$. Thus we get a compatible triple (ω, J, g) on T^*M .

The metric g is obviously complete. Indeed, define

$$\begin{aligned}\Sigma_e &:= \Sigma \cup (\cup_{s \geq 1} \psi_{s\sharp}(\Sigma \cap T^*M|_{\partial\Omega})) \\ \Sigma_t &:= \varphi_t(\Sigma_e).\end{aligned}$$

Identifying $\cup_{t \geq 0} \Sigma_t$ with $\Sigma_e \times [0, \infty)$, the metric g has the form

$$(12) \quad g(\cdot, \cdot) = e^t(g(\partial_t, \partial_t)dt^2 + g|_{\Sigma_e})((\varphi_t^{-1})_*\cdot, (\varphi_t^{-1})_*\cdot) \circ \varphi_t^{-1}.$$

It is clear the integral curves $\varphi_t(x)$, for $t > 0$ and $x \in \Sigma_e$, are minimizing geodesics of g . The distance from x to $\varphi_t(x)$, $L_x(t) = \int_0^t (e^t g(\partial_t, \partial_t))^{\frac{1}{2}} dt$, goes to ∞ as $t \rightarrow \infty$. Let $|s_1 - s_2|$ be positive and small. Assume $x \in T^*M|_{\partial\Lambda_{s_1}} \cap \Sigma_{t_1}$ and $y \in T^*M|_{\partial\Lambda_{s_2}} \cap \Sigma_{t_2}$. If $|t_2 - t_1| \rightarrow \infty$, $\text{dist}(x, y) \rightarrow \infty$. Let $\gamma(s)$ be a curve with $\gamma(0) = x$, $\gamma(1) = y$. From (12), we have the length $L(\gamma(s))$ of $\gamma(s)$ equals $e^{\frac{t_0}{2}} L(\varphi_{t_0}^{-1}(\gamma(s)))$, where $L(\varphi_{t_0}^{-1}(\gamma(s)))$ is the length of $\varphi_{t_0}^{-1}(\gamma(s))$. Thus we can get $\text{dist}(T^*M|_{\partial\Lambda_{s_1}}, T^*M|_{\partial\Lambda_{s_2}})$ is determined by the compact parts of $T^*M|_{\partial\Lambda_{s_1}}$ and $T^*M|_{\partial\Lambda_{s_2}}$. Thus

$$\text{dist}(T^*M|_{\partial\Lambda_{s_1}}, T^*M|_{\partial\Lambda_{s_2}}) > 0.$$

From equation (11), we know a curve $\gamma(t)$ from $T^*M|_{\partial\Lambda_{s_1}}$ to $T^*M|_{\partial\Lambda_{s_2}}$ has the same length with the curve $\psi_{s\sharp}(\gamma(t))$ from $T^*M|_{\partial\Lambda_{s+s_1}}$ to $T^*M|_{\partial\Lambda_{s+s_2}}$. $\psi_{s\sharp}$ is a symplectomorphism. Thus we have

$$\text{dist}(T^*M|_{\partial\Lambda_{s+s_1}}, T^*M|_{\partial\Lambda_{s+s_2}}) = \text{dist}(T^*M|_{\partial\Lambda_{s_1}}, T^*M|_{\partial\Lambda_{s_2}}).$$

Therefore, every bounded subset of T^*M is contained in a compact subset and is relatively compact. By Hopf-Rinow Theorem, this is equivalent to completeness.

From the Lemma 1 in [6] and the definition of the metric (9), it follows that the sectional curvature of g goes to zero as $x \rightarrow \infty$ in U . Thus the sectional curvature of g is bounded from above on $T^*M|_\Omega$. From (11) we know the sectional curvature

of g on $T^*M|_\Lambda$ is determined by the sectional curvature of g on $T^*M|_\Omega$ which is bounded from above. We get that the sectional curvature of g is bounded from above on T^*M .

Define $\Sigma' := \varphi_1(\Sigma \cup (\cup_{s \leq 2} \psi_{s\#}(\Sigma \cap T^*M|_{\partial\Omega})))$. Let W, U' denote the closure of the bounded and unbounded part of the complement to Σ' in $T^*M|_{\Omega \cup h(\partial\Omega \times [1,2])}$ respectively. Since W is compact, we know the injectivity radius of it is bounded away from zero. By Lemma 5.4, we know a curve $\gamma_s(x)$ through $x \in U_{\frac{1}{2}}, U_{\frac{1}{2}} = \cup_{t \geq \frac{1}{2}} \Sigma_t$, is a geodesic if and only if $\varphi_t^{-1}(\gamma_s(x))$ is a geodesic through $\varphi_t^{-1}(x)$, for any $0 < t \leq t_0(x)$, where $t_0(x)$ is the real number such that $\varphi_{t_0}^{-1}(x) \in \Sigma_{\frac{1}{2}}$. A curve $\gamma_t(x)$ is a geodesic through $x \in T^*M|_{\Lambda_2}$ if and only if $\psi_{s\#}^{-1}(\gamma_t(x))$ is a geodesic through $\psi_{s\#}^{-1}(x)$, for any $0 < s \leq s_0(x)$, $s_0(x)$ is the real number such that $\psi_{s_0\#}^{-1}(x) \in T^*M|_{\partial\Lambda_{\frac{3}{2}}}$. Thus if $\gamma(x)$ is a geodesic loop with small length $L(\gamma)$ through $x \in T^*M \setminus W$, there is a geodesic loop γ' in W with length $L(\gamma') \leq L(\gamma)$. Combined with the completeness and the upper bound of sectional curvature, we know if the injectivity radius of metric g on T^*M is not positive there is a geodesic loop γ' in W with length as small as we want (Lemma 16 in [18] P.142). This contradicts to the fact that the injectivity radius on W is positive. Thus we have the injectivity radius of g is bounded away from zero on T^*M . \square

The calculus of the geodesics is given in the following lemma.

Lemma 5.4. *With the metric g and notations defined in Proposition 5.3, a curve $\gamma_s(x)$ through $x \in U_{\frac{1}{2}}, U_{\frac{1}{2}} = \cup_{t \geq \frac{1}{2}} \Sigma_t$, is a geodesic if and only if $\varphi_t^{-1}(\gamma_s(x))$ is a geodesic through $\varphi_t^{-1}(x)$, for any $0 < t \leq t_0(x)$, where $t_0(x)$ is the real number such that $\varphi_{t_0}^{-1}(x) \in \Sigma_{\frac{1}{2}}$. A curve $\gamma_t(x)$ is a geodesic through $x \in T^*M|_{\Lambda_{\frac{3}{2}}}$ if and only if $\psi_{s\#}^{-1}(\gamma_t(x))$ is a geodesic through $\psi_{s\#}^{-1}(x)$, for any $0 < s \leq s_0(x)$, where $s_0(x)$ is the real number such that $\psi_{s_0\#}^{-1}(x) \in T^*M|_{\partial\Lambda_{\frac{3}{2}}}$.*

Proof. We only give the proof of the first assertion here, since the second can be proved similarly. Let $\pi : T^*M \rightarrow M$ be the projection of the cotangent bundle. Assume $x \in U_{\frac{1}{2}}$ such that $\pi(x) \in \Omega$. The metric is defined by $(\varphi_t)^*g = e^t g$. Thus we have

$$\begin{aligned} g &= (\varphi_t^{-1})^*(e^t g) \\ g(\cdot, \cdot) &= e^t g((\varphi_t^{-1})_* \cdot, (\varphi_t^{-1})_* \cdot) \circ \varphi_t^{-1}. \end{aligned}$$

Choose a local coordinate chart (V, φ) of M such that $\pi(x) \in V$ and $(\pi^{-1}(V), h')$ is a local trivialization of T^*M , i.e.

$$\pi^{-1}(V) \rightarrow V \times \mathbb{R}^n \rightarrow \varphi(V) \times \mathbb{R}^n$$

is a local coordinate chart of x in T^*M . Let $(x^1, \dots, x^n, y^1, \dots, y^n)$ be the local coordinates and denote

$$\partial_i = \frac{\partial}{\partial x^i} \quad i = 1, 2, \dots, n, \quad \partial_{\bar{i}} = \frac{\partial}{\partial y^i} \quad i = 1, 2, \dots, n.$$

Let

$$\begin{aligned} g_{ij} &= g(\partial_i, \partial_j), & g_{\bar{i}, \bar{j}} &= g(\partial_{\bar{i}}, \partial_{\bar{j}}), \\ g_{i, \bar{j}} &= g(\partial_i, \partial_{\bar{j}}), & g_{\bar{i}, j} &= g(\partial_{\bar{i}}, \partial_j). \end{aligned}$$

Then

$$\begin{pmatrix} (g^{ij}) & (g^{i\bar{j}}) \\ (g^{\bar{i}j}) & (g^{\bar{i}\bar{j}}) \end{pmatrix} = \begin{pmatrix} (g_{ij}) & (g_{i\bar{j}}) \\ (g_{\bar{i}j}) & (g_{\bar{i}\bar{j}}) \end{pmatrix}^{-1}.$$

The Christoffel symbols corresponding to the Riemannian metric g is given by

$$\Gamma_{ij}^k = \frac{1}{2} g^{k\bar{\ell}} (\partial_j g_{i\bar{\ell}} + \partial_i g_{j\bar{\ell}} - \partial_{\bar{\ell}} g_{ij}) + \frac{1}{2} g^{k\bar{\ell}} (\partial_j g_{i\bar{\ell}} + \partial_i g_{j\bar{\ell}} - \partial_{\bar{\ell}} g_{ij}).$$

The push forward of the vector fields can be given by

$$(\varphi_t^{-1})_* \begin{pmatrix} \partial_1 \\ \vdots \\ \partial_n \\ \partial_{\bar{1}} \\ \vdots \\ \partial_{\bar{n}} \end{pmatrix} \Big|_x = \begin{pmatrix} Id & 0 \\ 0 & e^{-t} Id \end{pmatrix} \begin{pmatrix} \partial_1 \\ \vdots \\ \partial_n \\ \partial_{\bar{1}} \\ \vdots \\ \partial_{\bar{n}} \end{pmatrix} \Big|_{\varphi_t^{-1}(x)}.$$

Then we have

$$\begin{pmatrix} (g_{ij}) & (g_{i\bar{j}}) \\ (g_{\bar{i}j}) & (g_{\bar{i}\bar{j}}) \end{pmatrix} = \begin{pmatrix} (e^t g_{ij}) & (g_{i\bar{j}}) \\ (g_{\bar{i}j}) & (e^{-t} g_{\bar{i}\bar{j}}) \end{pmatrix} \circ \varphi_t^{-1}.$$

Thus

$$\begin{pmatrix} (g^{ij}) & (g^{i\bar{j}}) \\ (g^{\bar{i}j}) & (g^{\bar{i}\bar{j}}) \end{pmatrix} = \begin{pmatrix} (e^{-t} g^{ij}) & (g^{i\bar{j}}) \\ (g^{\bar{i}j}) & (e^t g^{\bar{i}\bar{j}}) \end{pmatrix} \circ \varphi_t^{-1}.$$

To get the relation of Γ_{ij}^k with $\Gamma_{ij}^k \circ \varphi_t^{-1}$, we need the following relation

$$\begin{aligned} \partial_i g_{jk} &= \partial_i g(\partial_j, \partial_k) \\ &= \partial_i [e^t g((\varphi_t^{-1})_* \partial_j, (\varphi_t^{-1})_* \partial_k) \circ \varphi_t^{-1}] \\ &= e^t [(\partial_i g_{jk}) \circ \varphi_t^{-1} \partial_i (\varphi_t^{-1})^l + (\partial_{\bar{l}} g_{jk}) \circ \varphi_t^{-1} \partial_i (\varphi_t^{-1})^{\bar{l}}] \\ &= e^t (\partial_i g_{jk}) \circ \varphi_t^{-1}. \end{aligned}$$

Similarly, we have

$$\begin{aligned} \partial_i g_{\bar{j}k} &= (\partial_i g_{\bar{j}k}) \circ \varphi_t^{-1}, & \partial_i g_{j\bar{k}} &= (\partial_i g_{j\bar{k}}) \circ \varphi_t^{-1}, \\ \partial_{\bar{i}} g_{jk} &= (\partial_{\bar{i}} g_{jk}) \circ \varphi_t^{-1}, & \partial_i g_{\bar{j}\bar{k}} &= e^{-t} (\partial_i g_{\bar{j}\bar{k}}) \circ \varphi_t^{-1}, \\ \partial_{\bar{i}} g_{\bar{j}k} &= e^{-t} (\partial_{\bar{i}} g_{\bar{j}k}) \circ \varphi_t^{-1}, & \partial_{\bar{i}} g_{j\bar{k}} &= e^{-t} (\partial_{\bar{i}} g_{j\bar{k}}) \circ \varphi_t^{-1}, \\ \partial_{\bar{i}} g_{\bar{j}\bar{k}} &= e^{-2t} (\partial_{\bar{i}} g_{\bar{j}\bar{k}}) \circ \varphi_t^{-1}. \end{aligned}$$

The Christoffel symbols Γ_{ij}^k and $\Gamma_{ij}^{\bar{k}} \circ \varphi_t^{-1}$ have the following relation

$$\begin{aligned}
\Gamma_{ij}^k &= \frac{1}{2} g^{k\xi} (\partial_j g_{i\xi} + \partial_i g_{j\xi} - \partial_\xi g_{ij}) + \frac{1}{2} g^{k\bar{\xi}} (\partial_j g_{i\bar{\xi}} + \partial_i g_{j\bar{\xi}} - \partial_{\bar{\xi}} g_{ij}) \\
&= \frac{1}{2} e^{-t} g^{k\xi} \circ \varphi_t^{-1} (e^t (\partial_j g_{i\xi}) \circ \varphi_t^{-1} + e^t (\partial_i g_{j\xi}) \circ \varphi_t^{-1} - e^t (\partial_\xi g_{ij}) \circ \varphi_t^{-1}) \\
&\quad + \frac{1}{2} g^{k,\bar{\xi}} \circ \varphi_t^{-1} ((\partial_j g_{i\bar{\xi}}) \circ \varphi_t^{-1} + (\partial_i g_{j\bar{\xi}}) \circ \varphi_t^{-1} - (\partial_{\bar{\xi}} g_{ij}) \circ \varphi_t^{-1}) \\
&= \frac{1}{2} g^{k\xi} \circ \varphi_t^{-1} ((\partial_j g_{i\xi}) \circ \varphi_t^{-1} + (\partial_i g_{j\xi}) \circ \varphi_t^{-1} - (\partial_\xi g_{ij}) \circ \varphi_t^{-1}) \\
&\quad + \frac{1}{2} g^{k,\bar{\xi}} \circ \varphi_t^{-1} ((\partial_j g_{i\bar{\xi}}) \circ \varphi_t^{-1} + (\partial_i g_{j\bar{\xi}}) \circ \varphi_t^{-1} - (\partial_{\bar{\xi}} g_{ij}) \circ \varphi_t^{-1}) \\
&= \Gamma_{ij}^k \circ \varphi_t^{-1}.
\end{aligned}$$

Similarly, we have

$$\begin{aligned}
\Gamma_{ij}^{\bar{k}} &= e^t \Gamma_{ij}^{\bar{k}} \circ \varphi_t^{-1}, \quad \Gamma_{\bar{i}\bar{j}}^k = e^{-t} \Gamma_{\bar{i}\bar{j}}^k \circ \varphi_t^{-1} \\
\Gamma_{ij}^k &= e^{-t} \Gamma_{ij}^k \circ \varphi_t^{-1}, \quad \Gamma_{\bar{i}\bar{j}}^{\bar{k}} = \Gamma_{\bar{i}\bar{j}}^{\bar{k}} \circ \varphi_t^{-1} \\
\Gamma_{ij}^{\bar{k}} &= \Gamma_{ij}^{\bar{k}} \circ \varphi_t^{-1}, \quad \Gamma_{\bar{i}\bar{j}}^k = e^{-2t} \Gamma_{\bar{i}\bar{j}}^k \circ \varphi_t^{-1} \\
\Gamma_{\bar{i}\bar{j}}^{\bar{k}} &= e^{-t} \Gamma_{\bar{i}\bar{j}}^{\bar{k}} \circ \varphi_t^{-1}.
\end{aligned}$$

Now suppose $\gamma_s(x)$ is a curve through x . In local coordinates $\gamma_s(x)$ is given by

$$\gamma_s(x) = (\gamma^1(s), \dots, \gamma^n(s), \gamma^{\bar{1}}(s), \dots, \gamma^{\bar{n}}(s)).$$

Then $\varphi_t^{-1}(\gamma_s(x))$ is given by

$$\varphi_t^{-1}(\gamma_s(x)) = (\gamma^1(s), \dots, \gamma^n(s), e^{-t} \gamma^{\bar{1}}(s), \dots, e^{-t} \gamma^{\bar{n}}(s)).$$

Equation of geodesics in the local coordinates

$$\begin{aligned}
&\frac{d^2 \gamma^k}{ds^2} + \Gamma_{ij}^k \circ \varphi_t^{-1} \frac{d\gamma^i}{ds} \frac{d\gamma^j}{ds} + \Gamma_{ij}^k \circ \varphi_t^{-1} e^{-t} \frac{d\gamma^{\bar{i}}}{ds} \frac{d\gamma^{\bar{j}}}{ds} \\
&\quad + \Gamma_{ij}^k \circ \varphi_t^{-1} \frac{d\gamma^i}{ds} e^{-t} \frac{d\gamma^{\bar{j}}}{ds} + \Gamma_{ij}^k \circ \varphi_t^{-1} e^{-t} \frac{d\gamma^{\bar{i}}}{ds} e^{-t} \frac{d\gamma^{\bar{j}}}{ds} \\
&= \frac{d^2 \gamma^k}{ds^2} + \Gamma_{ij}^k \frac{d\gamma^i}{ds} \frac{d\gamma^j}{ds} + \Gamma_{ij}^k \frac{d\gamma^{\bar{i}}}{ds} \frac{d\gamma^{\bar{j}}}{ds} + \Gamma_{ij}^k \frac{d\gamma^i}{ds} \frac{d\gamma^{\bar{j}}}{ds} + \Gamma_{ij}^k \frac{d\gamma^{\bar{i}}}{ds} \frac{d\gamma^{\bar{j}}}{ds} \\
&= 0.
\end{aligned}$$

We know $\gamma_s(x)$ is a geodesic if and only if $\varphi_t^{-1}(\gamma_s(x))$ is a geodesic.

Now if $x \in U_{\frac{1}{2}}$ such that $\pi(x) \in \Lambda$, we can prove the first assertion in a similar way. Indeed, from equation (10) we know

$$\begin{aligned}(\psi_{s\#})^* \varphi_t^* g &= \varphi_t^* (\psi_{s\#})^* g, \\(\psi_{s\#})^* \varphi_t^* g &= e^t (\psi_{s\#})^* g, \\ \varphi_t^* g &= e^t g, \\ g &= e^t (\varphi_t^{-1})^* g.\end{aligned}$$

□

From Corollary 5.1 and Proposition 5.3, it is easy to get Corollary 1.3

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